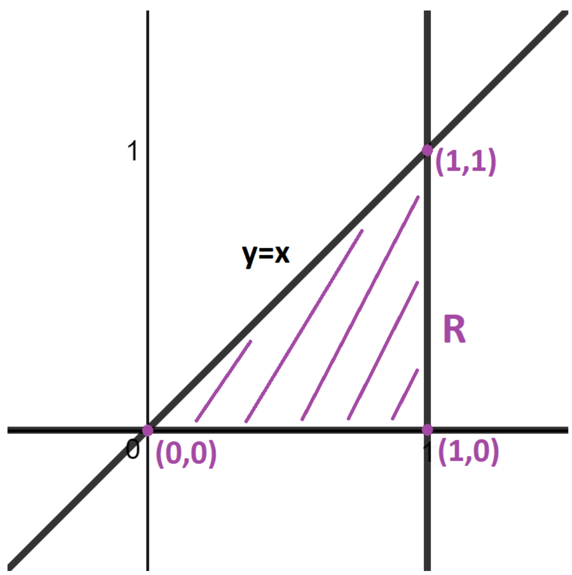
QUIZ 22 SOLUTIONS: LESSONS 29 & 30
NOVEMBER 14, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Find the average value of $f(x, y) = 4x^2y^3$ over the triangle with vertices

$$(0, 0), \quad (1, 0), \quad (1, 1).$$

Solution: We sketch a picture of the region:



The line which connects $(0, 0)$ and $(1, 1)$ is the line $y = x$. Hence, our region is described by

$$\begin{array}{ccccc} 0 & \leq & y & \leq & x \\ 0 & \leq & x & \leq & 1 \end{array}$$

Further, we are asked for the average of the function $f(x, y) = 4x^2y^3$ which means we need to find the area of R. R is half of a square of length and height equal to 1. Thus, the area of R is $1/2$. We write

$$\begin{aligned} \text{Average Value} &= \frac{1}{\text{Area of } R} \iint_R 4x^2y^3 \, dA \\ &= \frac{1}{\frac{1}{2}} \int_0^1 \int_0^x 4x^2y^3 \, dy \, dx \end{aligned}$$

$$= 2\int_0^1 \int_0^x 4x^2y^3\,dy\,dx$$

$$= 2\int_0^1 x^2y^4\Big|_{y=0}^{y=x}\,dx$$

$$= 2\int_0^1 x^2x^4\,dx$$

$$= 2\int_0^1 x^6\,dx$$

$$= \frac{2}{7}x^7\Big|_0^1$$

$$= \boxed{\frac{2}{7}}$$

2. [5 pts] Put the following matrix into row-echelon form:

$$\left[\begin{array}{rrr|r} 1 & 1 & 0 & 2 \\ 1 & 2 & 1 & -1 \\ -1 & 3 & -4 & 0 \end{array}\right]$$

Show your work and label each row operation you use.

<u>**Solution**</u>: There are several different ways of putting this matrix into row-echelon form. I outline one path below:

$$\xrightarrow{-R_1+R_2\to R_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & -3 \\ -1 & 3 & -4 & 0 \end{array}\right] \xrightarrow{R_1+R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & -3 \\ 0 & 4 & -4 & 2 \end{array}\right]$$

$$\xrightarrow{-4R_2+R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & -8 & 14 \end{array}\right] \xrightarrow{-R_3/8\to R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & -7/4 \end{array}\right]$$

Note that any matrix of the form

$$\left[\begin{array}{ccc|c} 1 & a & b & (13-5a-7b)/4 \\ 0 & 1 & c & (-5-7c)/4 \\ 0 & 0 & 1 & -7/4 \end{array}\right]$$

is also correct.